5.3.2. Metoda supstitucije za određene integrale

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Fotografija: Roma Kaiuk

Metoda supstitucije za određene integrale

Neka su zadane funkcije $f : D_1 \subseteq \mathbb{R} \to \mathbb{R}$ i $g : D_2 \subseteq \mathbb{R} \to \mathbb{R}$ takve da je funkcija $f(g(x)) \cdot g'(x)$ definirana i neprekidna na segmentu $[a, b] \subseteq \mathbb{R}$. Tada možemo računati

$$\int_a^b f(g(x)) \cdot g'(x)\,\mathrm{d}x = \begin{bmatrix} t = g(x) & a \mapsto g(a) \\ \mathrm{d}t = g'(x)\,\mathrm{d}x & b \mapsto g(b) \end{bmatrix} = \int_{g(a)}^{g(b)} f(t)\,\mathrm{d}t.$$

Metoda supstitucije za određene integrale

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Dokaz. Obje su strane gornje jednakosti jednake

$$F(g(x))\Big|_a^b,$$

gdje je F antiderivacija od f (dakle $F' = f$).

Primjer 1

Izračunajte integral $\int_1^2 (3x+4)^3 \, \mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
$$\int \sin x \, \mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Primjer 1

Izračunajte integral $\int_1^2 (3x+4)^3\,\mathrm{d}x$.

Rješenje. $\int_1^2 (3x+4)^3\,\mathrm{d}x$

$$= \begin{bmatrix} t = 3x+4 & 1 \mapsto 3\cdot 1 + 4 = 7 \\ \mathrm{d}t = 3\,\mathrm{d}x \;\Rightarrow\; \mathrm{d}x = \frac{\mathrm{d}t}{3} & 2 \mapsto 3 \cdot 2 + 4 = 10 \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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Rješenje. $\int_1^2 (3x+4)^3 \, dx$

$$= \begin{bmatrix} t = 3x+4 & 1 \mapsto 3\cdot 1 + 4 = 7 \\ dt = 3\, dx \;\Rightarrow\; dx = \frac{dt}{3} & 2 \mapsto 3 \cdot 2 + 4 = 10 \end{bmatrix}$$

$$= \int_7^{10} t^3 \cdot \frac{dt}{3}$$

$$\int dx = x + C$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

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$$= \int_7^{10} t^3 \cdot \frac{\mathrm{d}t}{3}$$

$$= \frac{1}{3}\cdot\frac{t^4}{4}\Big|_7^{10}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$= \int_7^{10} t^3 \cdot \frac{dt}{3}$$

$$= \frac{1}{3} \cdot \frac{t^4}{4} \Big|_7^{10}$$

$$= \frac{10^4 - 7^4}{12}$$

$$\int dx = x + C$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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$$= \frac{10^4 - 7^4}{12}$$

$$= \frac{2533}{4}.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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Zadatak 48(a)

Izračunajte integral $\int_0^{\frac{\pi}{4}} \sin^2 x \cdot \cos x \, \mathrm{d}x$.

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Rješenje. $\int_0^{\frac{\pi}{4}} \sin^2 x \cdot \cos x \, \mathrm{d}x$

$$= \begin{bmatrix} t = \sin x & 0 \mapsto \sin 0 = 0 \\ \mathrm{d}t = \cos x \, \mathrm{d}x & \frac{\pi}{4} \mapsto \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \end{bmatrix}$$

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Rješenje. $\int_0^{\frac{\pi}{4}} \sin^2 x \cdot \cos x \, \mathrm{d}x$

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$$= \int_0^{\frac{1}{\sqrt{2}}} t^2 \, \mathrm{d}t$$

$$= \frac{t^3}{3} \bigg|_0^{\frac{1}{\sqrt{2}}}$$

$$\int \mathrm{d}x = x + C$$
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$$= \int_0^{\frac{1}{\sqrt{2}}} t^2 \, \mathrm{d}t$$

$$= \frac{t^3}{3} \Big|_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{3} \left(\left(\frac{1}{\sqrt{2}} \right)^3 - 0^3 \right)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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$$= \int_0^{\frac{1}{\sqrt{2}}} t^2 \, \mathrm{d}t$$

$$= \frac{t^3}{3} \Big|_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{3} \left(\left(\frac{1}{\sqrt{2}} \right)^3 - 0^3 \right) = \frac{1}{6\sqrt{2}}.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(b)

Izračunajte integral $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
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Zadatak 48(b)

Izračunajte integral $\displaystyle\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$.

Rješenje. $\displaystyle\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$

$$= \left[\begin{array}{ccc} t = x^2 & & 1 \mapsto 1^2 = 1 \\ \mathrm{d}t = 2x\,\mathrm{d}x & \Rightarrow \quad x\,\mathrm{d}x = \frac{\mathrm{d}t}{2} & 2 \mapsto 2^2 = 4 \end{array}\right]$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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Izračunajte integral $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$.

Rješenje. $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$

$$= \left[\begin{array}{rcl} t = x^2 & & 1 \mapsto 1^2 = 1 \\ \mathrm{d}t = 2x\,\mathrm{d}x \;\Rightarrow\; x\,\mathrm{d}x = \frac{\mathrm{d}t}{2} & & 2 \mapsto 2^2 = 4 \end{array}\right]$$

$$= \int_1^4 7^t \cdot \frac{\mathrm{d}t}{2}$$

$$\int \mathrm{d}x = x + C$$

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Izračunajte integral $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$.

Rješenje. $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$

$$= \left[\begin{matrix} t = x^2 & & 1 \mapsto 1^2 = 1 \\ \mathrm{d}t = 2x\,\mathrm{d}x \;\Rightarrow\; x\,\mathrm{d}x = \frac{\mathrm{d}t}{2} & & 2 \mapsto 2^2 = 4 \end{matrix}\right]$$

$$= \int_1^4 7^t \cdot \frac{\mathrm{d}t}{2}$$

$$= \frac{1}{2} \cdot \frac{7^t}{\ln 7}\Big|_1^4$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(b)

Izračunajte integral $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$.

Rješenje. $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$

$$= \begin{bmatrix} & t = x^2 & 1 \mapsto 1^2 = 1 \\ \mathrm{d}t = 2x\,\mathrm{d}x & \Rightarrow \quad x\,\mathrm{d}x = \frac{\mathrm{d}t}{2} & 2 \mapsto 2^2 = 4 \end{bmatrix}$$

$$= \int_1^4 7^t \cdot \frac{\mathrm{d}t}{2}$$

$$= \frac{1}{2} \cdot \frac{7^t}{\ln 7}\bigg|_1^4$$

$$= \frac{1}{2\ln 7}\left(7^4 - 7^1\right)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(b)

Izračunajte integral $\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x$.

Rješenje.

$$\int_1^2 x \cdot 7^{x^2}\,\mathrm{d}x = \left[\begin{matrix} t = x^2 & & & 1 \mapsto 1^2 = 1 \\ \mathrm{d}t = 2x\,\mathrm{d}x & \Rightarrow & x\,\mathrm{d}x = \frac{\mathrm{d}t}{2} & 2 \mapsto 2^2 = 4 \end{matrix}\right]$$

$$= \int_1^4 7^t \cdot \frac{\mathrm{d}t}{2}$$

$$= \frac{1}{2} \cdot \frac{7^t}{\ln 7}\Big|_1^4$$

$$= \frac{1}{2\ln 7}\left(7^4 - 7^1\right)$$

$$= \frac{1197}{\ln 7}.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Izračunajte integral $\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(c)

Izračunajte integral $\displaystyle\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$.

Rješenje. $\displaystyle\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$

$$= \left[\begin{array}{ll} t = e^x + 7 & 0 \mapsto e^0 + 7 = 8 \\ \mathrm{d}t = e^x\,\mathrm{d}x & \ln 2 \mapsto e^{\ln 2} + 7 = 9 \end{array}\right]$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(c)

Izračunajte integral $\displaystyle\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$.

Rješenje.

$$\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x = \begin{bmatrix} t = e^x + 7 & 0 \mapsto e^0 + 7 = 8 \\ \mathrm{d}t = e^x\,\mathrm{d}x & \ln 2 \mapsto e^{\ln 2} + 7 = 9 \end{bmatrix} = \int_8^9 \frac{\mathrm{d}t}{\sqrt[4]{t}}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(c)

Izračunajte integral $\displaystyle\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$.

Rješenje.

$$\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x = \begin{bmatrix} t = e^x + 7 & 0 \mapsto e^0 + 7 = 8 \\ \mathrm{d}t = e^x\,\mathrm{d}x & \ln 2 \mapsto e^{\ln 2} + 7 = 9 \end{bmatrix} = \int_8^9 \frac{\mathrm{d}t}{\sqrt[4]{t}} = \int_8^9 t^{-\frac{1}{4}}\,\mathrm{d}t$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(c)

Izračunajte integral $\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$.

Rješenje.

$$\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x = \begin{bmatrix} t = e^x+7 & 0 \mapsto e^0+7=8 \\ \mathrm{d}t = e^x\,\mathrm{d}x & \ln 2 \mapsto e^{\ln 2}+7=9 \end{bmatrix} = \int_8^9 \frac{\mathrm{d}t}{\sqrt[4]{t}} = \int_8^9 t^{-\frac{1}{4}}\,\mathrm{d}t = \left.\frac{t^{\frac{3}{4}}}{\frac{3}{4}}\right|_8^9$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
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$$\int \cos x\,\mathrm{d}x = \sin x + C$$
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$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 48(c)

Izračunajte integral $\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x$.

Rješenje.

$$\int_0^{\ln 2} \frac{e^x}{\sqrt[4]{e^x+7}}\,\mathrm{d}x = \begin{bmatrix} t = e^x + 7 & 0 \mapsto e^0 + 7 = 8 \\ \mathrm{d}t = e^x\,\mathrm{d}x & \ln 2 \mapsto e^{\ln 2} + 7 = 9 \end{bmatrix} = \int_8^9 \frac{\mathrm{d}t}{\sqrt[4]{t}} = \int_8^9 t^{-\frac{1}{4}}\,\mathrm{d}t = \left.\frac{t^{\frac{3}{4}}}{\frac{3}{4}}\right|_8^9 = \frac{4}{3}\left(9^{\frac{3}{4}} - 8^{\frac{3}{4}}\right).$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$